Higher-order parametric level statistics in disordered systems

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Higher-order parametric level correlations in disordered systems with broken time-reversal symmetry are studied by mapping the problem onto a model of coupled Hermitian random matrices. Closed analytical expression is derived for a parametric density-density correlation function that corresponds to a perturbation of disordered system by a multicomponent flux. [S1063-651X(99)14503-3]

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Parametric level statistics reflect the response of the spectrum $\{E_n\}$ of complex chaotic systems to an external perturbation. A few years ago it was shown [1–3] that a system whose spectrum follows closely the universal fluctuations predicted by the random matrix theory [4] should also exhibit a universal parametric behavior. This conclusion was reached by analyzing the dimensionless autocorrelator of level velocities of electron in a disordered metallic sample with a ring topology, enclosing a magnetic flux φ. A diagrammatic perturbation technique was used in the range of fluxes $g^{-1/2} \ll \varphi \ll 1$ [1], while the opposite limit, $\varphi < g^{-1/2}$ [2], has been treated within the framework of the supersymmetry formalism [5,6]. [Here $g \gg 1$ is the dimensionless conductance.] In this particular problem, the parametric correlations take a universal form involving the rescaled parameter $X^2 = 4\pi g \varphi^2$, with $g = E_c/\Delta$, the ratio of the Thouless energy and the mean level spacing. Numerical simulations have supported the point that the universal character of parametric level statistics extends to a wider class of chaotic systems without disorder [chaotic billiards] whose Hamiltonian depends on some external parameter x. In such systems, the spectral fluctuations taken at different values of x become system—independent after rescaling, $x \to X$, which involves solely the ‘‘generalized’’ dimensionless conductance $g = (4\pi)^{-1}\Delta^{-2}\langle[\partial E_n(x)/\partial x]^2\rangle$. Along with the diagrammatic technique and the supersymmetry formalism, the parametric correlations have been studied in detail within the model of Brownian motion [7,8], in the semiclassical limit [9,10], and by the loop-equations technique [11].

A further burst of activity in the field occurred after it was realized [3,7,12–15] that the problem of parametric level correlations is identical to the ground-state dynamics of the integrable many-body quantum model known as Calogero-Sutherland-Moser [CSM] system. This gave new information about CSM space-time (r,τ) correlation functions that can be obtained from parametric density-density correlators $\langle \nu(E,0)\nu(E',\varphi)\rangle$ involving only two different external parameters by mapping [3] $X^2 \to -2i\tau, E/\Delta \to r$. For the more general situation of higher-order correlation functions the connection between CSM fermions and quantum chaotic systems has been established [15] as well by using the supersymmetry technique; however, it has not led to any explicit analytical results beyond the two-point correlators due to enormous increase of number of entries in the supermatrix fields, thereby making any explicit calculations in that approach impossible. Extensions to higher-order statistics can be performed by using an involved method of differential equations for quantum correlation functions proposed in the much earlier work [16].

In the present paper we address the issue of higher-order parametric level statistics within the framework of the random matrix theory, by appealing to the model of coupled Hermitian random matrices [17]. The latter enables us to provide a complete information about parametric correlations of single electron level densities in the presence of the multicomponent flux perturbing a disordered system, characterized by a dimensionless conductance $g \gg 1$. To the best of our knowledge, this is the first detailed study of higher-order parametric level statistics in disordered systems that adopts the conventional language of the random matrix theory.

In what follows we consider a weakly disordered system fallen in the universal (metallic) regime, $g \gg 1$, which is known [5] to be modeled by invariant ensembles of large random matrices. Assuming that the time reversal symmetry is completely broken (unitary symmetry), one can statistically describe an unperturbed single electron spectrum by a Gaussian unitary ensemble [GUE] of large $N \times N$ random matrices H_0 distributed in accordance with the probability density $\mathcal{P}[H_0] \propto \exp\{-\mathrm{Tr} H_0^2\}$. Such a distribution $\mathcal{P}[H_0]$ induces the energy scale Δ being the mean level spacing, $\Delta = \pi(2N)^{-1/2}$. Let us now apply a Gaussian perturbation consisting of d components $\vec{\varphi}_d = (\phi_1, \ldots, \phi_d)$, which does not change the global unitary symmetry, and which drives the Hamiltonian H_0 to $H = H_0 + \sum_{k=1}^{d} \phi_k H_k$, with matrices H_k drawn from GUE: $\mathcal{P}[H_k] \propto \exp\{-\mathrm{Tr}\, H_k^2\}$ for $k = 1, \ldots, d$. This choice corresponds to the equal ‘‘strength’’ of each component of the ‘‘vector’’ perturbation $\vec{\varphi}_d$ since the average $\langle (H_k)_{\mu\nu}(H_k)_{\mu'\nu'}\rangle$ is independent of the index k. The quantity that provides the most detailed information about parametric correlations in the case of the multicomponent perturbation $\vec{\varphi}_d$ is the correlator of level densities $\nu(E,\vec{\varphi}_\sigma) = \mathrm{Tr}\, \delta(E - H_0 - \sum_{k=1}^{\sigma} \phi_k H_k)$ taken at both different values of energy E and of σ. For this reason, we will concentrate on the dimensionless multipoint correlator

$$k_{p_0,\ldots,p_d}(\{\omega^{(0)}\},\vec{0};\ldots;\{\omega^{(d)}\},\vec{\boldsymbol{\varphi}}_d) = \Delta^m\Bigg\langle \prod_{i_0=1}^{p_0}\nu(E+\omega_{i_0}^{(0)},\vec{0})\prod_{i_1=1}^{p_1}\nu(E+\omega_{i_1}^{(1)},\vec{\boldsymbol{\varphi}}_1)\cdots \times\prod_{i_d=1}^{p_d}\nu(E+\omega_{i_d}^{(d)},\vec{\boldsymbol{\varphi}}_d)\Bigg\rangle, \tag{1}$$

where $m=p_0+\cdots+p_d$, and the angular brackets stand for averaging over ensembles of Hermitian matrices H_k with $k=0,\ldots,d$. Equation (1) can be rewritten as a $(d+1)$ multiple matrix integral over matrices $\tilde{H}_0=H_0$ and $\tilde{H}_{\sigma>0}=H_0+\Sigma_{k=1}^{\sigma}\phi_k H_k$,

$$k_{p_0,\ldots,p_d}\propto\int d\tilde{H}_0\cdots\int d\tilde{H}_d\prod_{\sigma=0}^{d}\prod_{i_\sigma=1}^{p_\sigma}\mathrm{Tr}\,\delta(E+\omega_{i_\sigma}^{(\sigma)}-\tilde{H}_\sigma) \times\exp\Bigg\{-\mathrm{Tr}\Bigg[\sum_{\alpha=0}^{d}(\phi_\alpha^{-2}+\phi_{\alpha+1}^{-2})\tilde{H}_\alpha^2 -2\sum_{\alpha=0}^{d-1}\phi_{\alpha+1}^{-2}\tilde{H}_\alpha\tilde{H}_{\alpha+1}\Bigg]\Bigg\}, \tag{2}$$

with $\phi_0=1$ and $\phi_{d+1}=\infty$. [This convention is relaxed everywhere below Eq. (9).] We notice that the strengths $\phi_k(k=1,\ldots,d)$ of the perturbation are supposed to be small, $\phi_k\ll 1$. This is justified in the thermodynamic limit $N\to\infty$, since for Gaussian perturbation accepted above, ϕ_k are known to scale with N as $\phi_k=\pi N^{-1/2}X_k$, with X_k being the set of dimensionless parameters of order unity [18].

Our crucial observation is that Eq. (2) can be interpreted as a density-density correlator in the effective model of $(d+1)$ Hermitian random matrices coupled in a chain: Each matrix $\tilde{H}_\alpha$ is represented by a point, and two adjacent matrices $\tilde{H}_\alpha$ and $\tilde{H}_{\alpha+1}$ are joined by a line if the coupling of the type $\exp\{c_\alpha \mathrm{Tr}\,\tilde{H}_\alpha\tilde{H}_{\alpha+1}\}$ is present in Eq. (2). In this situation, the joint probability density of eigenvalues of all the matrices in the chain can be deduced through the Itzykson-Zuber integral [19] making the model of random Hermitian matrices coupled in a chain to be a completely solvable. In accordance with the Eynard-Mehta theorem [17], the dimensionless correlator $k_{p_0,\ldots,p_d}$ can be represented as a determinant of the $m\times m$ block matrix, $m=p_0+\cdots+p_d$, consisting of $(d+1)\times(d+1)$ rectangular submatrices $K_{\alpha,\beta}$ with $\alpha,\beta=1,\ldots,(d+1)$, each of them having $p_{\alpha-1}\times p_{\beta-1}$ entries [20],

$$k_{p_0,\ldots,p_d}=\mathrm{Det}\begin{pmatrix} [K_{1,1}(\omega_{i_0}^{(0)},\omega_{j_0}^{(0)})]_{p_0\times p_0} & [K_{1,2}(\omega_{i_0}^{(0)},\omega_{j_1}^{(1)})]_{p_0\times p_1} & \cdots & [K_{1,d+1}(\omega_{i_0}^{(0)},\omega_{j_d}^{(d)})]_{p_0\times p_d} \\ [K_{2,1}(\omega_{i_1}^{(1)},\omega_{j_0}^{(0)})]_{p_1\times p_0} & [K_{2,2}(\omega_{i_1}^{(1)},\omega_{j_1}^{(1)})]_{p_1\times p_1} & \cdots & [K_{2,d+1}(\omega_{i_1}^{(1)},\omega_{j_d}^{(d)})]_{p_1\times p_d} \\ \vdots & \vdots & \ddots & \vdots \\ [K_{d+1,1}(\omega_{i_d}^{(d)},\omega_{j_0}^{(0)})]_{p_d\times p_0} & [K_{d+1,2}(\omega_{i_d}^{(d)},\omega_{j_1}^{(1)})]_{p_d\times p_1} & \cdots & [K_{d+1,d+1}(\omega_{i_d}^{(d)},\omega_{j_d}^{(d)})]_{p_d\times p_d} \end{pmatrix}. \tag{3}$$

The matrix kernels $K_{\alpha,\beta}$ in Eq. (3) are

$$K_{\alpha,\beta}(\xi,\eta)=\Delta[H_{\alpha,\beta}(\xi,\eta)-E_{\alpha,\beta}(\xi,\eta)], \tag{4}$$

where

$$H_{\alpha,\beta}(\xi,\eta)=\sum_{j=0}^{N-1}\frac{1}{h_j}Q_{\alpha,j}(\xi)P_{\beta,j}(\eta), \tag{5}$$

and

$$E_{\alpha,\beta}(\xi,\eta)=(w_\alpha*\cdot*w_{\beta-1})(\xi,\eta) \tag{6}$$

for $1\leqslant\alpha<\beta\leqslant d+1$; otherwise, $E_{\alpha,\beta}=0$. Here the partial weights w_α are

$$w_\alpha(\xi,\eta)=\exp\left(-\frac{V_\alpha(\xi)+V_{\alpha+1}(\eta)}{2}+2\phi_\alpha^{-2}\xi\eta\right), \tag{7a}$$

$$V_\alpha(\xi)=(\phi_{\alpha-1}^{-2}+\phi_\alpha^{-2})[\delta_{\alpha,1}+\delta_{\alpha,d+1}+1]\xi^2 \tag{7b}$$

[compare with the weight of the matrix model, Eq. (2)]. The notation $(w_\alpha*\cdot*w_{\beta-1})(\xi,\eta)$ stands for the product of the partial weights w integrated over internal variables of that product. Two sets of orthogonal functions $P_{\alpha,j}$ and $Q_{\beta,j}$ entering Eq. (5) are determined recursively,

$$P_{\alpha,j}(\xi)=\int d\eta P_{\alpha-1,j}(\eta)w_{\alpha-1}(\eta,\xi), \tag{8a}$$

$$Q_{\beta,j}(\xi)=\int d\eta w_\beta(\xi,\eta)Q_{\beta+1,j}(\eta), \tag{8b}$$

for $2\leqslant\alpha\leqslant d+1$ and $1\leqslant\beta\leqslant d$; the starting points of the recursions (8a) and (8b) are the polynomials $P_{1,j}=P_j$ and $Q_{d+1,j}=Q_j$ orthogonal with respect to a *nonlocal* weight $W(\xi,\eta)=(w_1*\cdot*w_d)(\xi,\eta)$,

$$\int d\xi\int d\eta P_i(\xi)W(\xi,\eta)Q_j(\eta)=h_j\delta_{ij}. \tag{9}$$

Close inspection of the equations above shows that the basic orthogonal polynomials P_j and Q_j can be expressed in terms of Hermite polynomials, $P_j(\xi)=H_j(\xi),Q_j(\xi)=H_j(\xi[1+\Sigma_{k=1}^{d}\phi_k^2]^{-1/2})$. Then, step-by-step integrations in Eqs. (8) yield

$$P_{\alpha,j}(\xi)=\frac{\prod_{k=1}^{\alpha-1}(\phi_k\sqrt{\pi})}{\left[1+\sum_{k=1}^{\alpha-1}\phi_k^2\right]^{j/2}}e^{-F_\alpha(\xi)}\Phi_j\left(\frac{\xi}{C_{\alpha-1}}\right), \quad (10a)$$

$$Q_{\alpha,j}(\xi)=\frac{\prod_{k=\alpha}^{d}(\phi_k\sqrt{\pi})}{\left[1+\sum_{k=\alpha}^{d}\phi_k^2\right]^{j/2}}e^{F_\alpha(\xi)}\Phi_j\left(\frac{\xi}{C_{\alpha-1}}\right), \quad (10b)$$

where we have introduced the Hermite functions [21] $\Phi_j(\xi)=\exp[-\xi^2/2]H_j(\xi)$. Also, we defined the function

$$F_\alpha(\xi)=\frac{\xi^2}{2}[C_{\alpha-1}^{-2}+(\phi_\alpha^{-2}-\phi_{\alpha-1}^{-2})], \quad (11)$$

and the constant $C_\alpha=[1+\Sigma_{k=1}^{\alpha}\phi_k^2]^{1/2}$. [In order to compactify the formulas, it is agreed from now on that $\phi_{d+1}=\phi_d, \phi_0=\phi_1$, $\Sigma_{k=\alpha}^{\beta<\alpha}(\cdots)=0$, and $\Pi_{k=\alpha}^{\beta<\alpha}(\cdots)=1$]. One can verify that the orthogonality relation (9) is satisfied with

$$h_j=2^j j!\sqrt{\pi}\left[1+\sum_{k=1}^{d}\phi_k^2\right]^{-j/2}\prod_{k=1}^{d}(\phi_k\sqrt{\pi}), \quad (12)$$

so that the first term in Eq. (4) is

$$H_{\alpha,\alpha}(\xi,\eta)=e^{F_\alpha(\xi)-F_\alpha(\eta)}\sum_{j=0}^{N-1}\Phi_j\left(\frac{\xi}{C_{\alpha-1}}\right)\Phi_j\left(\frac{\eta}{C_{\alpha-1}}\right), \quad (13a)$$

$$H_{\alpha<\beta}(\xi,\eta)=\prod_{k=\alpha}^{\beta-1}(\phi_k\sqrt{\pi})e^{F_\alpha(\xi)-F_\beta(\eta)} \times\sum_{j=0}^{N-1}\frac{\Phi_j\left(\frac{\xi}{C_{\alpha-1}}\right)\Phi_j\left(\frac{\eta}{C_{\beta-1}}\right)}{\left[1+\sum_{k=\alpha}^{\beta-1}\phi_k^2\right]^{j/2}}, \quad (13b)$$

$$H_{\alpha>\beta}(\xi,\eta)=\frac{1}{\prod_{k=\beta}^{\alpha-1}(\phi_k\sqrt{\pi})}e^{F_\alpha(\xi)-F_\beta(\eta)} \times\sum_{j=0}^{N-1}\frac{\Phi_j\left(\frac{\xi}{C_{\alpha-1}}\right)\Phi_j\left(\frac{\eta}{C_{\beta-1}}\right)}{\left[1+\sum_{k=\beta}^{\alpha-1}\phi_k^2\right]^{-j/2}}. \quad (13c)$$

The second term in Eq. (4) is found from Eqs. (6) and (7),

$$E_{\alpha,\beta}(\xi,\eta)=\frac{\prod_{k=\alpha}^{\beta-1}(\phi_k\sqrt{\pi})\mathrm{e}^{G_\alpha(\xi)-G_\beta(\eta)}}{\sqrt{\pi\sum_{k=\alpha}^{\beta-1}\phi_k^2}}\exp\left\{-\frac{(\xi-\eta)^2}{\sum_{k=\alpha}^{\beta-1}\phi_k^2}\right\} \quad (14)$$

for $\beta\geqslant\alpha+2$, while $E_{\alpha,\alpha}=0$ and $E_{\alpha,\alpha+1}=w_\alpha$. Here the function G_α reads

$$G_\alpha(\xi)=\frac{\xi^2}{2}(\phi_\alpha^{-2}-\phi_{\alpha-1}^{-2}). \quad (15)$$

Now, we are in position to compute the matrix kernels $K_{\alpha,\beta}$ via Eqs. (4), (13), and (14) in the leading order in $N\to\infty$ and keeping $X_k=\phi_k N^{1/2}/\pi\sim O(1)$ fixed. The simplest, diagonal kernel $K_{\alpha,\alpha}$ can be evaluated through the Christoffel-Darboux formula [22], supplemented by the asymptotics of Hermite functions,

$$\left\{\begin{matrix}\Phi_{2N}(t)\\ \Phi_{2N+1}(t)\end{matrix}\right\}\simeq\frac{(-1)^N}{N^{1/4}\sqrt{\pi}}\left\{\begin{matrix}\cos(2tN^{1/2})\\ \sin(2tN^{1/2})\end{matrix}\right\} \quad (16)$$

where $t\sim\Delta O(N^0)$. One obtains,

$$K_{\alpha,\alpha}(\xi,\eta)=e^{G_\alpha(\xi)-G_\alpha(\eta)}\frac{\sin[\pi\Delta^{-1}(\xi-\eta)]}{\pi\Delta^{-1}(\xi-\eta)}. \quad (17)$$

Two other cases, $\alpha<\beta$ and $\alpha>\beta$, demand more effort. For $\alpha<\beta$ we represent the sum for $H_{\alpha<\beta}$ in Eq. (13b) as a difference of two series, $\Sigma_{j=0}^{\infty}(\cdots)-\Sigma_{j=N}^{\infty}(\cdots)$. The first sum is exactly computable by making use of the Mehler summation formula [22]. In the thermodynamic limit, this procedure yields a term that is equal to $E_{\alpha,\beta}$ in Eq. (14), and therefore it gets canceled from the expression (4) for $K_{\alpha<\beta}$, which is completely due to the remaining sum $\Sigma_{j=N}^{\infty}(\cdots)$. To evaluate the latter, we replace the sum over j by an integral to get

$$K_{\alpha<\beta}(\xi,\eta)=-\prod_{k=\alpha}^{\beta-1}(\phi_k\sqrt{\pi})e^{G_\alpha(\xi)-G_\beta(\eta)} \times\int_1^\infty d\lambda_1\cos\left\{\pi\frac{\xi-\eta}{\Delta}\lambda_1\right\} \times\exp\left(-\frac{\pi^2\lambda_1^2}{2}\sum_{k=\alpha}^{\beta-1}X_k^2\right). \quad (18)$$

In the case $\alpha>\beta$ the large-j terms in Eq. (13c) yield the main contribution to the sum due to the factor $[1+\Sigma_{k=\beta}^{\alpha-1}\phi_k^2]^{j/2}$. Then, passing from summation to integration, we derive

$$K_{\alpha>\beta}(\xi,\eta)=\frac{1}{\prod_{k=\beta}^{\alpha-1}(\phi_k\sqrt{\pi})}e^{G_\alpha(\xi)-G_\beta(\eta)} \times\int_0^1 d\lambda\cos\left(\pi\frac{\xi-\eta}{\Delta}\lambda\right)\exp\left(\frac{\pi^2\lambda^2}{2}\sum_{k=\beta}^{\alpha-1}X_k^2\right). \quad (19)$$

Notice that the structure of the block matrix in Eq. (3) allows one to simultaneously suppress the prefactors of the form $\Pi_k(\cdots)e^{(\cdots)}$ in Eqs. (17), (18), and (19). Having this in mind, we come down to the closed analytical determinantal expression Eq. (3) for $(p_0+\cdots+p_d)$-point density-density correlator with $K_{\alpha,\beta}$ replaced by $M_{\alpha,\beta}$,

$$M_{\alpha,\alpha}(\xi,\eta)\equiv\frac{\sin[\pi\Delta^{-1}(\xi-\eta)]}{\pi\Delta^{-1}(\xi-\eta)}, \quad (20a)$$

$$M_{\alpha<\beta}(\xi,\eta)\equiv-\int_1^\infty d\lambda_1\,\cos\left(\pi\frac{\xi-\eta}{\Delta}\lambda_1\right)\times\exp\left(-\frac{\pi^2\lambda_1^2}{2}\sum_{k=\alpha}^{\beta-1}X_k^2\right), \quad (20b)$$

$$M_{\alpha>\beta}(\xi,\eta)\equiv\int_0^1 d\lambda\,\cos\left(\pi\frac{\xi-\eta}{\Delta}\lambda\right)\exp\left(\frac{\pi^2\lambda^2}{2}\sum_{k=\beta}^{\alpha-1}X_k^2\right). \quad (20c)$$

Equations (3) and (20) are the main result of the paper. They provide a detailed information about higher order parametric density-density correlations in the case of multiparameter perturbation of disordered system. Several particular correlators can be readily deduced from our general expression: (i) For the scalar perturbation, one obtains that $k_{p,q}=\Delta^{p+q}\langle\prod_{i=1}^{p}\nu(E+\omega_i,0)\prod_{j=1}^{q}\nu(E+\Omega_j,\phi)\rangle$ is determined by

$$k_{p,q}\equiv\mathrm{Det}\begin{pmatrix} M_{\alpha,\alpha}(\omega_i,\omega_j) & M_{\alpha<\beta}(\omega_i,\Omega_j)\\ M_{\alpha>\beta}(\Omega_i,\omega_j) & M_{\alpha,\alpha}(\Omega_i,\Omega_j)\end{pmatrix}, \quad (21)$$

where $M_{\alpha,\beta}$ are those given by Eqs. (20) with $\Sigma_k X_k^2\to X^2$; (ii) By replacement [3] $\omega_i/\Delta\to r_i,\Omega_i/\Delta\to R_i$ and $X^2\to -2i\tau$ in Eq. (21) one arrives at the space-time correlation function in the CSM model with a coupling $\lambda=1$; here the coordinates $\{r_i\}$ correspond to the time $t=0$, while the $\{R_i\}$ refer to the time $t=\tau$.

In summary, we presented a random-matrix-theory treatment of the problem of higher-order parametric spectral statistics in disordered systems with broken time reveral symmetry in the presence of the multiparameter perturbation. A complete analytical solution was based on the mapping of the initial problem onto a model of random Hermitian matrices coupled in a chain. As a particular case of the general solution, given by Eqs. (3) and (20) the multipoint parametric spectral correlator Eq. (21) for the scalar perturbation has been obtained. Together with a well-established correspondence between CSM fermions and parametric level statistics, the latter expression provides an information about the space-time correlation function in the Calogero-Sutherland-Moser model of free, noninteracting fermions.

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